EXPANSION

Summary:

(i) *To expand is to remove the brackets from an expression. Thus in expanded form* ***a(x + 5) = ax + 5a***

(ii) *To multiply two brackets, the first bracket is multiplied by each term in the second bracket*

EXAMPLES:

1. *Remove the brackets and simplify :*

(i) $a(3x + 5) - x\left(a - \frac{2a}{x}\right)$ ***(ii)(x + 5)(x + 3)*** ***(iii) (3x + 2)(x– 4)***

(iv) 5(x – 3)(3x – 2) ***(v)*** $(x - 5)^2$ ***(vi)*** $(2x + 3)^2$ ***(vii)*** $a\left(1 - \frac{ax}{2}\right)^2$

(viii) $\left(x + \frac{1}{x}\right)^2$ ***(ix)*** $\left(x - \frac{1}{x}\right)^2$ ***(x)*** $\left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right)$

(xi) $(a + b)^2$ ***(xii)*** $(a - b)^2$ ***(xiii)*** $(a + b)(a - b)$

QUADRATIC IDENTITIES

Summary:

The following expansions result into:

(i) $(a + b)^2 = a^2 + 2ab + b^2$

(ii) $(a - b)^2 = a^2 - 2ab + b^2$

(iii) $(a + b)(a - b) = a^2 - b^2$

EXAMPLES:

1.*Use the result* $(a + b)^2 = a^2 + 2ab + b^2$, *to evaluate the following:*

(i) $(102)^2$ ***(ii)*** $(10 \cdot 3)^2$ ***(iii)*** $(201)^2$

Soln:

(i) *If* $(a + b)^2 = a^2 + 2ab + b^2$,

$$\Rightarrow (102)^2 = (100 + 2)^2 = 100^2 + 2(100)(2) + 2^2$$
$$= 10000 + 400 + 4$$
$$= 10404$$

2.*Use the result* $(a - b)^2 = a^2 - 2ab + b^2$, *to evaluate the following:*

(i) $(99)^2$ ***(ii)*** $(9 \cdot 7)^2$ ***(iii)*** $(91)^2$

Soln:

(i) *If* $(a - b)^2 = a^2 - 2ab + b^2$,

$$\Rightarrow (99)^2 = (100 - 1)^2 = 100^2 - 2(100)(1) + 1^2$$
$$= 10000 - 200 + 1$$
$$= 9801$$

3. *If* $x^2 + 6x - 91 = (x + a)^2 + b$, *find the values of* ***a*** *and* ***b***

4. *Express* $x^2 + 6x - 12$ *in the form* $(x + a)^2 + b$

5. *Express* $3x^2 + 9x - 30$ *in the form* $a(x + b)^2 + c$.

6. *Given that* $x + y = 5$ *and* $xy = 4$, *find the values of:*

(i) $x^2 + y^2$

(ii) $x - y$

7. *If* $x + \frac{1}{x} = 3$, *find the values of:*

(i) $x^2 + \frac{1}{x^2}$ ***(ii)*** $x - \frac{1}{x}$

8. *Find the coefficient of* x^2 *in the expansion of* $(2x - 5)^3$

9. *Find the coordinates of point* ***P*** *on the* ***x****–axis which is equidistant from the points* ***A(5, 9)*** *and* ***B(–4, 6)***

10. *Find the equation of the locus of points equidistant from point* ***P(6, 0)*** *and* ***Q(–8, 4)***

EER:

1. *Expand the following expressions:*

(i) (2x + 5)(x + 3) ***(ii) (3x + 2)(4x– 7)*** ***(iii) 3(x – 4)(5x – 3)***

(iv) $(x + y)^2$ ***(v)*** $(x - y)^2$ ***(vi)*** $(x + y)(x - y)$

2. *Make* ***P*** *the subject in the given formula* $p^2 = (p - q)(p - r)$

4. *Expand the expression* $\left(p + \frac{1}{p}\right)^2$

5. *Expand the expression* $h\left(1 - \frac{2}{3}h^3\right)^2$

6. *Simplify the expression* $\left(x + \frac{1}{x}\right)^2 - \left(x - \frac{1}{x}\right)^2$

7. (i) *Write down the expansion of* $(a + b)^2$

(ii) *Use the result in* ***(i)*** *above to evaluate* $(10 \cdot 6)^2$

8. *Expand and simplify* $(3 - x)^3$

9. *Express* $2x^2 - 7x - 30$ *in the form* $a(x + b)^2 + c$.

10. *If* $3x^2 + 9x - 30 = a(x + b)^2 + c$, *find the vales of* ***a, b*** *and* ***c***

11. *Given that* $x^2 + y^2 = 29$ *and* $\boldsymbol{x + y = 3}$, *find the values of:*

(i) $x^2 + 2xy + y^2$

(ii) $2xy$

(iii) $x^2 - 2xy + y^2$

(iv) $\boldsymbol{x - y}$, *hence solve for* ***x*** *and* ***y***

12. *If* $x^2 + \frac{1}{x^2} = 27$, *find the values of* $x - \frac{1}{x}$

13. *Express* $2x^2 - 7x - 30$ *in the form* $a(x + b)^2 + c$.

14. *If* $(3x - 4)^2 = 9x^2 + ax + b$, *find the vales of* ***a*** *and* ***b***

15. *If* $x + y = 8$ *and* $\boldsymbol{xy = 10}$, *find the value of* $x^2 + y^2$

16. *If* $(x - b)^2 = x^2 - 20x + a$, *find the vales of* ***a*** *and* ***b***

17. *If* $4x^2 - 32x + k - 20 = a(x + b)^2$, *find the value of* ***k***

18. *Find the coordinates of point* ***P*** *on the* ***y****–axis which is equidistant from the points* ***A(–5, –2)*** *and* ***B(3, 2)***

19. *A point* ***P(x, y)*** *moves such that its distance from the origin,* ***O*** *is equal to its distance from the point* ***Q(1, 2).*** *Find the locus (equation) relating* ***x***

and ***y.***

FACTORING BY GROUPING

Summary:

1. ***(i)*** *To factorise is to write the expression as a product of its factors. Thus in factorised form* $3x^2 + 6x = 3x(x + 2)$ *and* $8ab^2 - 12a^2b = 4ab(2b - 3a)$

(ii) *To factorise by grouping, an expression must have four terms*

2. *This method of factoring is performed as follows:*

(i) *Factor out the* ***HCF*** *if any*

(ii) *Group the terms with common factors*

(iii) *Factor each group*

EXAMPLES:

1. *Factorise the following expressions completely:*

(i) ah + ak + bh + bk ***(ii) 6mx – 3nx + 2my – ny***

(iii) $6x^2y + 2xy^2 - 27x - 9y$ ***(iv) 6ah – 3ak + bk – 2bh***

(vi) 2ab – 3 + 2a – 3b ***(vii)*** $2b^3 + 3a^3 + 3ab^2 + 2a^2b$

2. *Without using a calculator or tables, evaluate:*

(i) 0·25 × 2195 – 1795 × 0·25 ***(ii) 65 × 43·6 + 65 × 37·2 – 65 × 70·8***

(iii) $3 \cdot 142^2 - 3 \cdot 042 \times 3 \cdot 142$ ***(iv)*** $\frac{30 \cdot 25^2 - 30 \cdot 15 \times 30 \cdot 25}{0 \cdot 0025}$

(v) $\frac{21 \cdot 35 \times 41 \cdot 35 - 21 \cdot 35^2}{0 \cdot 02}$

EER:

1. *Factorise the following expressions completely:*

(i) $ac - 2ab + 3bc - 6b^2$ ***(ii)*** $3a^2 + 2ab - 12ac - 8bc$

(iii) $2a^3 - 2ab + b^3 - ab^2$ ***(iv)*** $4q^4 - 8q^3 + 12q^2 - 24q$

(v) $2a^2 + 4ab + 3ac + 6bc$ ***(vi)*** $4xy^3 - x^3 - yx^2 + 4y^3$

(vii) $3x^3 + 7x^2 + 12x + 28$ ***(viii)*** $5x + 10y - ax - 2ay$

2. *Without using a calculator or tables, evaluate:* $0{\cdot}35 \times 2595 - 2495 \times 0{\cdot}35$

3. *Use factors to evaluate:* $14 \times 398 - 198 \times 14$

4. *Use factors to evaluate:* $617 \times 793 + 786 \times 793 + 597 \times 793$

5. *Without using a table or a calculator, evaluate:* $3 \cdot 2 \times 758 - 658 \times 3 \cdot 2$

6. *By first removing the brackets, factorise completely*

$$3ph\left(\frac{1}{3p} - \frac{2q}{h}\right) - 6q\left(\frac{h}{3} - \frac{p}{2q}\right)$$

THE DIFFERENCE OF TWO SQUARES

Summary:

The result $a^2 - b^2 = (a + b)(a - b)$ *is called the difference of two squares*

EXAMPLES:

1. *Show that* $(x + y)(x - y) = x^2 - y^2$

2. *Factorise the following expressions completely:*

(i) $p^2 - q^2$ ***(ii)*** $x^2 - 9$ ***(iii)*** $x^2 - 1$ ***(iv)*** $25x^2 - 16$

(v) $9x^2 - 1$ ***(vi)*** $16x^2 - 25y^2$ ***(vii)*** $3x^2 - 27y^2$ ***(viii)*** $50x^3 - 18xy^2$

(ix) $8x^2 - 72$ ***(x)*** $x^2 - \frac{1}{x^2}$ ***(xi)*** $3x^2 - \frac{1}{3}$ ***(xii)*** $x^4 - y^4$

(xiii) $x^4 - 1$ ***(xiv)*** $ax^4 - a$ ***(xv)*** $x^4 - 16$ ***(xvi)*** $81x^5 - x$

3. *Factorise the following expressions completely:*

(i) $a^2 - b^2 + 2a + 2b$ ***(ii)*** $x^3 - 3x^2 - 16x + 48$ ***(iii)*** $18x^3 + 9x^2 - 2x - 1$

(iv) $2p^2q^3 - pq^3 + pq - 2p^2q$ ***(v)*** $(2x - 9)^2 - 25$ ***(vi)*** $(2x + 5)^2 - 81$

(vii) $(2x + 1)^2 - (x + 5)^2$

4. *Without using a calculator or tables, evaluate:*

(i) $251^2 - 250^2$ ***(ii)*** $7 \cdot 46^2 - 2 \cdot 54^2$ ***(iii)*** $5 \cdot 2 \times 3 \cdot 75^2 - 5 \cdot 2 \times 1 \cdot 25^2$

(iv) $67 \cdot 68^2 - 53 \cdot 6 \times 48 \cdot 69 - 32 \cdot 32^2 + 38 \cdot 69 \times 53 \cdot 6$ ***(v)*** $\frac{68 \cdot 75^2 - 31 \cdot 25^2}{3 \cdot 75}$

(vi) $\frac{6 \cdot 51^2 - 2 \cdot 81^2}{0 \cdot 932 \times 74}$ ***(vii)*** $\frac{32 \cdot 135^2 - 17 \cdot 865^2}{0 \cdot 7135}$

5. *Given that* $a^2 - b^2 = 63$ *and* $a + b = 21$, *find the values of:*

(i) $a - b$ ***(ii)*** a *and* b

6. *Given that* $x^2 - y^2 = 135$ *and* $x - y = 9$, *find the values of* x *and* y

7. *Given that* $x + y = 5$ *and* $\boldsymbol{xy = 4}$, *find the values of:*

(i) $x^2 + y^2$

(ii) $\boldsymbol{x - y}$

(iii) $x^2 - y^2$

8. *If* $x + \frac{1}{x} = 3$, *find the values of:*

(i) $x^2 + \frac{1}{x^2}$

(ii) $x - \frac{1}{x}$

(iii) $x^2 - \frac{1}{x^2}$

EER:

1. *Factorise completely* $27 - 3x^2$

2. Factorise completely $32x^2 - 18y^2$

3. Factorise completely $2a^4 - 162$

4. Factorise completely $x^3 - 3x^2 - x + 3$

5. Factorise completely $x^2 - y^2 + 3x + 3y$

6. Factorise completely $567 - 28y^2$

7. Factorise completely $3p^4 - 48$

8. By first removing the brackets, factorise completely **$y(ay - x) + x(y - ax)$**

9. Without using a calculator or tables, evaluate the following:

(i) $102^2 - 98^2$ **(ii)** $351^2 - 350^2$ **(iii)** $495^2 - 5^2$

(iv) $3 \cdot 1^2 - 1 \cdot 9^2$ **(v)** $7 \cdot 46^2 - 2 \cdot 54^2$ **(vi)**

10. Without using a calculator or tables, evaluate: $\frac{6 \cdot 25^2 - 3 \cdot 75^2}{0 \cdot 25}$

11. Without using a calculator or tables, evaluate: $\frac{30 \cdot 5^2 - 19 \cdot 5^2}{11}$

12. Factorise the following expressions completely:

(i) $4x^2 - 1$ **(ii)** $7 - 63x^2$ **(iii)** $9x^2 - 16$ **(iv)** $9x^2 - 4y^2$

(v) $4x^2 - 9$ **(vi)** $x^3 - xy^2$ **(vii)** $50x^3 - 18xy^2$ **(viii)** $3x^2 - 27y^2$

(ix) $x^2 - \frac{1}{9}$ **(x)** $a^2 - \frac{1}{4}$ **(xi)** $4x^2 - \frac{y^2}{100}$ **(xii)** $ax^4 - a$

(xiii) $a^4 - b^4$ **(xiv)** $x^2 - y^2 - x - y$ **(xv)** $x - x^2 + y + y^2$

13. Factorize $p^2 - q^2$, hence find the value of $7 \cdot 3^2 - 2 \cdot 7^2$

14. Given that $x^2 - y^2 = 16$ and $x + y = 9$, find the values of x and y

15. *Given that* $x^2 - y^2 = 33$ *and* $x - y = 3$, *find the values of* x *and* y

16. *Given that* $x^2 - y^2 = 24$ *and* $x + y = 12$, *find the values of* x *and* y

17 *Factorise completely* $2a^2 - 32$

18. *Factorise completely* $3x^3 - 12x$

19. *Factorise completely* $8a^3 - 18b^2$

20. *Factorise completely* $3x^3 + yx^2 - 12xy^2 - 4y^3$

21. *Factorise completely* $2x^3 - 50xy^2$

22. *Factorise completely* $9(x + y)^2 - x^2$

FACTORING QUADRATICS

Summary:

1. *A quadratic expression is written in the form* $ax^2 + bx + c$

2. *The following apply when factoring a quadratic expression* $ax^2 + bx + c$ *:*

(i) *Find two numbers which multiply to give* ***ac*** *and add up to* ***b***

(ii) *Replace* ***b*** *with the sum of those numbers*

(iii) *Factor the terms by grouping*

EXAMPLES:

1. *Factorise the following expressions completely:*

(i) $3x^2 + 11x + 6$ ***(ii)*** $5x^2 - 13x + 6$ ***(iii)*** $3x^2 - 4x - 15$

(iv) $x^2 + 2x - 15$ ***(v)*** $x^2 + 6x + 9$ ***(vi)*** $x^2 - 4x + 3$

(vii) $x^2 - 5x + 6$ **(vii)** $a^2 + 2ab + b^2$ **(ix)** $3x^2 - 4xy + y^2$

(x) $2y^2 - 3xy - 2x^2$ **(xi)** $20x^2y^2 + xy - 1$ **(xii)** $6p^2 + pq^2 - 2q^4$

2. *Factorise the following expressions completely:*

(i) $25 - (x^2 + 2xy + y^2)$ **(ii)** $a^2 + b^2 - 4 + 2ab$

(iii) $a^2 - 5a - 36 + ay + 4y$ **(iv)** $a^2 - 2ab - 5a + 2b + 4$

(v) $(x^2 - 5x)^2 - 36$

3. *Simplify the following fractions as far as possible:*

(i) $\frac{x^2 - 9}{5x^2 - 13x - 6}$ **(ii)** $\frac{18x^2 - 2}{3x^2 + 2x - 1}$ **(iii)** $\frac{x^2 + 8x + 15}{x^2 - 25}$

(iv) $\frac{3x^2 + x - 4}{5x^2 - 7x + 2}$ **(v)** $\frac{3x^2 - 4x - 15}{5x^2 - 9x - 18}$

4. *Express* $\frac{2x^2 + x - 6}{x^2 - 4} + \frac{1}{x - 2}$ *in the form* $\frac{ax + b}{cx + d}$

5. *Express* $\frac{2}{x + 4} + \frac{4}{x - 3} - \frac{4(x + 4)}{x^2 + x - 12}$ *in the form* $\frac{a}{x + b}$

6. *Express* $\frac{2x^2 + x}{4x^2 - 1} - \frac{2}{2x - 1}$ *in the form* $\frac{ax + b}{cx + d}$

7. *Express* $\frac{x + 1}{x + 2} - \frac{x + 4}{3x + 6}$ *in the form* $\frac{ax + b}{cx + d}$

EER:

1. *Factorise the following expressions completely:*

(i) $5x^2 - 11x + 2$ **(ii)** $x^2 + 2x - 15$ **(iii)** $x^2 + 8x + 16$

(iv) $25x^2 - 10x + 1$ **(v)** $x^2 - 5x + 6$ **(vi)** $a^2 - 2ab + b^2$

(vii) $9x^2 - 12x + 4$ **(viii)** $2x^3 - 2x^2 - 4x$ **(ix)** $2x^2 - 21xy - 50y^2$

(x) $x^2 - 15xy + 56y^2$ **(xi)** $12x^2 - 17xy + 5y^2$ **(ix)** $2x^2y^2 + 13xy + 15$

2. *By first simplifying the expression factorise completely*

$6y^2 - 11y - 6 - (2y - 3)^2$

3. *By first simplifying the expression factorise completely* $(4 - x)^2 - 2x$

4. *Factorise* $35 - 2a - a^2$

5. *Factorise* $120 - 7x - x^2$

6. *Factorise completely* $2x^3 + 5x^2y - 12xy^2$

7. *Factorise* $x^2y^2 - 8xy - 48$

8. *Factorise* $6x^2 + xy^2 - 2y^4$

9. *Factorise completely* $2x^4 - 7x^2 - 4$

10. *Factorise completely* $y^2 - 2yb - 5y + 2b + 4$

11. *Simplify the following fractions as far as possible:*

(i) $\frac{9x^2 - 1}{3x^2 + 2x - 1}$ **(ii)** $\frac{x^2 + 6x - 16}{x^2 - 4}$ **(iii)** $\frac{x^2 + x - 2}{x^2 - x - 6}$

(iv) $\frac{x^2 - 2x - 3}{x^2 - x - 2}$ **(v)** $\frac{a^2 - a + \frac{1}{4}}{a^2 - \frac{1}{4}}$ **(vi)** $\frac{3x^2 - 4xy + y^2}{9x^2 - y^2}$

(vii) $\frac{2y^2 - 3xy - 2x^2}{4y^2 - x^2}$ **(viii)** $\frac{6a - 3ab - 2a - a^2}{a^2 + ab}$ **(ix)** $\frac{9a^2y - 16b^2y^3}{4by^2 - 3ay}$

12. Simplify $\frac{x^2 + 2x}{x^3 - x^2 - 6x}$, hence solve $\frac{x^2 + 2x}{x^3 - x^2 - 6x} = \frac{1}{4}$

13. Express $\frac{4}{x+5} + \frac{1}{x-2} - \frac{7}{x^2+3x-10}$ in the form $\frac{a}{x + b}$

14. Express $\frac{2}{3x + 2} + \frac{5x + 3}{9x^2 - 4}$ in the form $\frac{ax+b}{cx^2 + d}$

15. Express $\frac{7}{x-2} - \frac{x + 5}{x^2 - 3x + 2}$ in the form $\frac{a}{x + b}$

16. Express $\frac{2}{x + 2} + \frac{8x + 4}{x^2 - 4}$ in the form $\frac{ax}{x^2 + b}$

SOLVING QUADRATIC BY FACTORING

Summary:

1. *A quadratic equation is written in the form* $ax^2 + bx + c = 0$

2. *A quadratic equation has two solutions often called the roots of the equation. It is possible for the two solutions to be the same*

3. *This method uses the fact that if* $m \times n = 0$, *then* $m = 0$ *or* $n = 0$

EXAMPLES:

1. *Solve the following equations:*

(i) $2x^2 + 7x + 6 = 0$ **(ii)** $x^2 - x = 12$ **(iii)** $x^2 + 12x + 36 = 0$

(iv) $x^2 - 5x = 0$ **(v)** $5x^2 = 3x$ **(vi)** $5x^3 - 5x^2 - 10x = 0$

(vii) $(x + 4)(x + 2) = 48$ **(vii)** $x + 2 = \frac{3}{x}$ **(ix)** $\frac{1}{x - 1} + \frac{4}{x} = 3$

(x) $\frac{1}{x+1} = 1 - \frac{5}{2x-4}$ **(xi)** $\frac{1}{x+2} + \frac{1}{x^2-4} = \frac{2}{5}$

(xii) $\frac{x-1}{x-2} + \frac{5x-13}{x^2-x-2} = 0$

2. Factorise $25x^2 - 64$, hence solve the equation $25x^2 - 64 = 0$

3. Factorise $(x^2 - 5x)^2 - 36$, hence solve the equation $(x^2 - 5x)^2 - 36 = 0$

4. Solve the following equations:

(i) $x^2 - 16 = 0$ **(ii)** $81x^2 - 64 = 0$ **(iii)** $(2x-9)^2 = 25$

(iv) $(2x+5)^2 - 81 = 0$ **(v)** $(2x+1)^2 - (x+5)^2 = 0$

SOLVING QUADRATIC BY FORMULA

Summary:

If $ax^2 + bx + c = 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

EXAMPLES:

1. Solve the following equations:

(i) $2x^2 + 7x + 6 = 0$ **(ii)** $3x^2 - 4x = 15$ **(iii)** $x^2 - x - 12 = 0$

(iv) $\frac{x}{x+1} - \frac{x+1}{3x-1} = \frac{1}{4}$

EER:

1. Factorise $x^2 + 6x - 91$, hence solve the equation $x^2 + 6x - 91 = 0$

2. Use the factorisation method to solve the equation: $3x^2 - 19x - 14 = 0$

3. Factorise $25x^2 - 36$, hence solve the equation $25x^2 - 36 = 0$

4. Given that $p \uparrow q = p^2 + 2pq - q$, find the values of:

(i) $2 \uparrow 9$

(ii) p if $p \uparrow 4 = 29$

5. If $\frac{x^2}{x+c} = \frac{u}{w}$, make x the subject of the formula

6. Solve the equation: $6x^2 + 5x = 6$

7. Solve the equation $2x - 7 = \frac{15}{x}$

8. Solve the equation: $\frac{x-1}{x+1} = \frac{7x-13}{6x+1}$

9. Solve the following equations:

(i) $x^2 - 3x = 0$ **(ii)** $x^2 - 5x + 6 = 0$ **(iii)** $8x^2 - 10x + 3 = 0$

(iv) $x^2 + 5x = 0$ **(v)** $x^2 - 9 = 0$ **(vi)** $25x^2 - 36 = 0$

(vii) $x^2 + 6x + 9 = 0$ **(viii)** $x^2 + 7x + 12 = 0$ **(ix)** $10x^2 + 19x + 6 = 0$

(x) $4x^2 - 12x + 9 = 0$ **(xi)** $x(x-1) = 6$ **(xii)**

$(2x+5)^2 - (x-2)^2 = 0$ **(xiii)** $(x-3)(x-2) = 12$

10. Solve the following equations:

(i) $x = \frac{10}{3+x}$ **(ii)** $\frac{x}{x+3} = \frac{8}{x+6}$ **(iii)** $\frac{x}{x+3} - \frac{2}{x} = 1$

(iv) $\frac{x-3}{2} = \frac{5}{x}$ **(v)** $x = \frac{15-x^2}{2}$ **(vi)** $\frac{1}{x-1} - \frac{1}{x^2-1} = \frac{1}{6}$

(vii) $\frac{x+1}{2x+5} = \frac{x-1}{3}$ ***(viii)*** $\frac{x}{2x-5} = \frac{3x}{x+11}$ ***(ix)*** $\frac{x}{x-1} - \frac{2}{x} = \frac{1}{x-1}$

(x) $\frac{1}{3x-4} + \frac{x}{x+1} = 1$ ***(xi)*** $\frac{x^2+4}{x^2-4} + \frac{2}{x+2} = \frac{5}{x-2}$

(xii) $x + 3 + \frac{3}{x-1} = \frac{4-x}{x-1}$

SOLVING QUADRATIC BY COMPLETING SQUARES

Summary:

(i) *A quadratic perfect square is written in the form* $a(x+b)^2$

(ii) *In completing squares, the original equation is first expressed in the form* $a(x+b)^2 + c = 0$ *and then solved*

EXAMPLES:

1. *Express* $x^2 - 8x - 20$ *in the form* $(x+a)^2 + b$. *Hence solve the equation* $x^2 - 8x - 20 = 0$.

2. *Express* $x^2 + 6x - 91$ *in the form* $(x+a)^2 + b$. *Hence solve the equation* $x^2 + 6x - 91 = 0$.

3. *Express* $2x^2 - 7x - 30$ *in the form* $a(x+b)^2 + c$. *Hence solve the equation* $2x^2 - 7x - 30 = 0$.

4. *Express* $3x^2 + 9x - 30$ *in the form* $a(x+b)^2 + c$. *Hence solve the equation* $3x^2 + 9x - 30 = 0$.

5. *Solve the equation* $3x^2 - 7x + 2 = 0$ *by the method of completing squares*

6. *Given that* $4x^2 - 12x + k$ *is a perfect square, find the value of* ***k***

EER:

1. *If* $x^2 + 8x - 20 = (x + a)^2 + b$, *find the values of* ***a*** *and* ***b***

QUADRATIC WORD POBLEMS

EXAMPLES:

1. *A rectangle of length* ***(3x + 1)cm*** *and width* ***(3x – 2)cm*** *has an area of* $130cm^2$. *Find the dimensions and perimeter of the rectangle*

2. *A man is* ***22*** *years older than his son. The product of their ages is* ***240*** *years. Find their present ages*

3. *Find the values of* ***x*** *for which the fraction* $\frac{5x + 7}{x^2 + 5x + 6}$ *is undefined*

4. *The distance between the point* ***(k, k+2)*** *and the origin is* ***10 units.*** *Find the possible values of* ***k***

5. *Find the value of* ***n*** *in the following equations:*

(i) $212_n = 25_{nine}$ ***(ii)*** $223_n = 63_{ten}$

6. *Solve the following simultaneous equations:*

(i) $2x - y = 5$, $xy = 12$ *(ii)* $6x + 7y = 4$, $x^2 + y^2 = 13$ *(iii)* $x + 3y = 10$, $3x^2 + 5y^2 = 16xy$

7. *Find the coordinates of the point of intersection of the line* ***y = 2 – 3x*** *and the*

curve $y = x^2 - 4x$

8. *Find the area of a right triangle whose hypotenuse is* ***13cm*** *long and its perimeter is* ***30cm.***

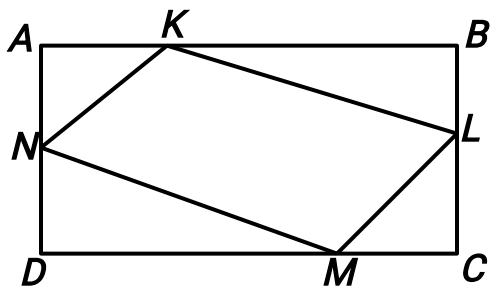
9. *The cost of* ***n*** *pens is* ***Shs 7,500.*** *This same amount of money can buy*

five extra pens if a discount of ***Shs 50*** *per pen is given. Find the cost of each pen*

10. *A basin can be filled by tap* ***P*** *in* ***10*** *minutes less than* ***Q.*** *If the two taps take* ***12*** *minutes to fill this basin when they are turned on at once, find the time taken by each tap separately to fill the basin.*

11. *A train takes two hours less for a journey of* ***300km*** *if its speed is increased by* $5kmh^{-1}$ *from its normal speed. Find its normal speed*

12. ***ABCD*** *is a rectangle in which* ***AB = 12cm, BC = 7cm*** *and* ***AK = BL = CM = DN = y cm.***



If the area of ***KLNM*** *is* $45cm^2$, *find the value of* ***y***

FORMING A QUADRATIC EQUATION

Summary:

The relation $x^2 - (\text{sum of roots})x + \text{product of roots} = 0$ *is used to form a quadratic equation whose roots are known*

EXAMPLES:

1. *Form a quadratic equation whose roots are* ***2*** *and* ***3***

2. *Form a quadratic equation whose roots are* ***–5*** *and* ***2***

3. *Form a quadratic equation whose solution set is* ***{–7, 3}***

4. *Form a quadratic equation whose roots are* ***–2*** *and* $\frac{1}{3}$

EER:

1. *The length of a rectangular floor is* **8** *metres longer than its width. If the area of the floor is* $65m^2$, *find the dimensions and perimeter of the floor.*

2. *If* $125_n = 85_{ten}$, *find the value of* **n**

3. *A number exceeds four times its reciprocal by* **3.** *Find the number.*

4. *Find the area of a right triangle whose hypotenuse is* **34cm** *long and its perimeter is* **80cm.**

5. *Find the number which when added to its square gives a total of* **42**

6. *Find the dimensions of the rectangle whose diagonal is* **10cm** *long and its length exceeds the width by* **2cm.**

7. *Find the dimension of a rectangle whose area is* $72cm^2$ *and its perimeter is* **34cm.**

8. *If* $124_n = 310_{four}$, *find the base that* **n** *represents*

9. *Express* $x^2 - x - \frac{3}{4}$ *in the form* $(x + p)^2 + q$. *Hence solve the equation* $x^2 - x - \frac{3}{4} = 0$.

10. *Solve for* **y** *in the following equations:*

(i) $27 \times 3^{2y} = \left(3^y\right)^y$

(ii) $(16)^{y^2} = 8^{4y-3}$

11. *Given that vector* $p = \begin{pmatrix} x \\ x+2 \end{pmatrix}$, *find the possible values of* **x** *for which* $|p| = 10$

12. *The distance between the points* **(4, 8)** *and* **(1, k)** *is* **5 units.** *Find the possible values of* **k**

13. *A rectangle of length* **(4x – 1)cm** *and width* **2x cm** *has an area of* $10cm^2$. *Find:*

(i) the value of ***x***
(ii) its length and width
(iii) its perimeter

14. *Find the coordinates of the points of intersection of the curve* $y = 5x^2 - 13$ *and the line* ***y = 7.***

15. *A right angled triangle of base* ***(x – 4)cm*** *and height* ***(x – 2)cm*** *has a hypotenuse of* ***xcm*** *long. Find:*

(i) the value of ***x***
(ii) its dimensions
(iii) its perimeter and area

16. *A right angled triangle of base* ***xcm*** *and height* ***(x – 7)cm*** *has a hypotenuse of* ***(x + 1)cm*** *long. Find:*

(i) the value of ***x***
(ii) its base and height
(iii) its perimeter and area

17. *Find the coordinates of the points of intersection of the curve* $y = x^2 - 5x$ *and the line* ***y + 6 = 0***

18. *Find the coordinates of the point of intersection of line* ***y – 5x + 9 = 0*** *and the curve* $y = x^2 - 3$

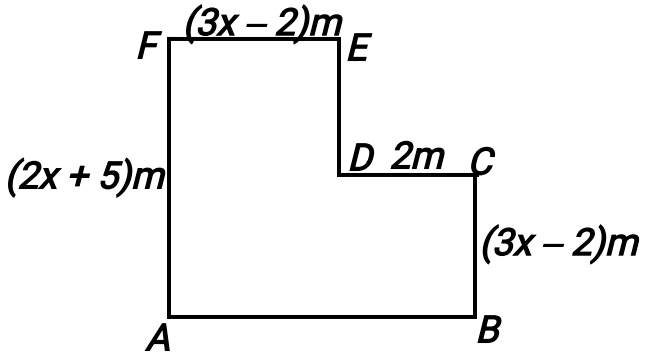
19. *Find the values of* ***x*** *for which the fraction* $\frac{5x + 7}{x^2 - x - 12}$ *is undefined*

20. *A group of people planned to contribute equally towards a water project which needed* ***Shs 2,000,000*** *to complete. However,* ***40*** *members of the group withdrew from the project. As a result, each of the remaining members was to contribute* ***Shs 2500*** *more.*
(a) *Find the original number of members in the group*
(b) *Forty five percent of the value of the project was donated by the Development Bank. Calculate the amount required to be contributed by each of the remaining members of the group*

(c) Members contribution was in terms of labour provided and money contributed. The ratio of the value of labour to the money contributed was 6:19, calculate the total amount of money contributed by the members

21. The figure below shows an L–shaped carpet ABCDEF in which CD = 2m,

BC = EF = (3x – 2)m and AF = (2x + 5)m.



Given that the area of the carpet is $25m^2$,

(a) Show that $6x^2 + 17x - 39 = 0$

(b) Calculate the:

(i) length of the longest side of the carpet

(ii) perimeter of the carpet

GRAPHING QUADRATIC CURVES

Summary:

1. (i) By drawing a suitable line, a quadratic graph can be used to solve related equations

(ii) The solution to the equation are the x–values at the point where the graphs meet

2. The appropriate line is obtained as follows:

(i) Compare the equation to that of the graph

(ii) Eliminate the term in x^2 *to obtain the line*

3. (i) *The curve* $y = ax^2 + bx + c$ *has a vertical line of symmetry whose equation is given by* $x = -\frac{b}{2a}$

(ii) The maximum or minimum value of the function occur at its turning points

EXAMPLES:

1. *Find the equation of the line which should be drawn on the graph* $y = x^2 + 2x - 4$ *to solve each of these equations:*

(i) $x^2 + 2x - 4 = 0$ ***(ii)*** $x^2 + 2x - 7 = 0$ ***(iii)*** $x^2 + x - 5 = 0$

(iv) $x^2 - x - 6 = 0$ ***(v)*** $2x^2 - 5x - 3 = 0$ ***(vi)*** $1 + x - 2x^2 = 0$

2. (a) *Draw a graph of* $y = x^2 - 4x + 3$ *for* $0 \leq x \leq 4$
(use a scale of ***2cm:1 unit*** *on both axes)*

(b) *Use your graph to solve the equations:*

(i) $x^2 - 4x + 3 = 0$

(ii) $x^2 - 5x + 4 = 0$

(c) *State the:*

(i) *equation of the line of symmetry*

(ii) *minimum value of* ***y***

(iii) *value of* ***x*** *at which the minimum value of* ***y*** *occurs*

(iv) *range of values of* ***x*** *for which* $x^2 - 4x + 3 < 0$

3. (a) *Draw a graph of* $y = x^2 - x - 6$ *for* $-3 \leq x \leq 4$

(use a scale of ***1cm:1 unit*** *on both axes)*

(b) *Use your graph to solve the equations:*

(i) $x^2 - x - 6 = 0$

(ii) $x^2 - x - 2 = 0$

(iii) $x^2 + x - 2 = 0$

(iv) $2x^2 - 5x - 3 = 0$

(c) *State the:*

(i) *equation of the line of symmetry*

(ii) *range of values of* ***x*** *for which* $x^2 - x - 6 < 0$

4. (a) *Copy and complete the table below for the function* $y = 3 - 3x - x^2$

x	***–5***	***–4***	***–3***	***–2***	***–1***	***0***	***1***	***2***
$-x^2$				***–4***				***–4***
$-3x$				***6***				***–6***
3				***3***				***3***
y				***5***				***–7***

(b) *Draw a graph of* $y = 3 - 3x - x^2$ *for* $-5 \leq x \leq 2$

(use a scale of ***1cm:1 unit*** *on both axes)*

(c) *Use your graph to solve the equations:*

(i) $3 - 3x - x^2 = 0$ ***(ii)*** $x^2 + 3x - 4 = 0$ ***(iii)*** $x^2 + 4x + 3 = 0$

(iv) $x^2 + 2x - 8 = 0$ ***(v)*** $2x^2 + x - 6 = 0$

5. (a) *Copy and complete the table below for the function* $y = (x - 2)(x + 1)$

x	***–3***	***–2***	***–1***	***0***	***1***	***2***	***3***	***4***
(x – 2)		***–4***		***–4***				***2***
(x + 1)		***–1***		***6***				***5***
y		***4***		***3***				***10***

(b) *Draw a graph of* $y = (x - 2)(x + 1)$ *for* $-3 \leq x \leq 4$
(use a scale of ***1cm:1 unit*** *on both axes)*

(c) *Use your graph to solve the equations:*

(i) $x^2 - x - 2 = 0$ ***(ii)*** $x^2 - 2x - 5 = 0$

(iii) $2x^2 - x - 15 = 0$

(d) *State the:*

(i) *equation of the line of symmetry*

(ii) *minimum value of the function*

(iv) *range of values of* ***x*** *for which* $(x - 2)(x + 1) < 0$

EER:

1. *Use the graphical method to solve the simultaneous equations* $y = 3x^2 - 3x$ *and* $y = 10 - 5x$ *for* $-3 \leq x \leq 3$.

2. (a) *Draw a graph of* $y = x^2 - 2x + 1$ *for* $-3 \leq x \leq 3$
(use a scale of ***1cm:1 unit*** *on both axes)*

(b) *Use your graph to solve the equations:*

(i) $x^2 - 2x + 1 = 0$

(ii) $x^2 - x - 6 = 0$

3. (a) *Copy and complete the table below for the function* $y = 2 + x - x^2$

X	***–3***	***–2***	***–1***	***0***	***1***	***2***	***3***	***4***
$-x^2$		***–4***					***–9***	
x		***–2***					***3***	
2		***2***					***2***	
Y		***–4***					***–4***	

(b) *Draw a graph of* $y = 2 + x - x^2$ *for* $-3 \leq x \leq 4$
(use a scale of ***1cm:1 unit*** *on both axes)*

(c) *Use your graph to solve the equations:*

(i) $x^2 - x - 6 = 0$

(ii) $x^2 - 3x - 4 = 0$

(iii) $3x^2 - 2x - 1 = 0$

4. *On the same axes, draw the graphs of* $y = 2x^2$ *and* $y = \frac{5x}{2} + 5$ *for* $-2 \leq x \leq 3$ *(use a scale of* ***2cm : 1 unit*** *on the* ***x-axis*** *and* ***1cm : 2 units*** *on the* ***y-axis)***

(b) *Use your graphs to solve the equations:*

(i) $4x^2 - 5x - 10 = 0$

(ii) $6x^2 + 10x - 30 = 0$ *correct to* ***2*** *decimal places*

5. (a) *Copy and complete the table below for the function* $y = (x - 1)(x - 3)$

X	***–1***	***0***	***1***	***2***	***3***	***4***	***5***
(x – 1)		***–1***			***2***		
(x – 3)		***–3***			***0***		
Y		***3***			***0***		

(b) *Draw a graph of* $y = (x - 2)(x + 1)$ *for* $-1 \leq x \leq 5$ *(use a scale of* ***1cm:1 unit*** *on both axes)*

(c) *Use your graph to solve the equations:*

(i) $x^2 - 4x + 3 = 0$

(ii) $x^2 - 4x + 1 = 0$

(d) *State the:*

(i) *minimum value of the function*

(iv) *range of values of* ***x*** *for which* $(x - 1)(x - 3) < 0$

6. (a) *On the same axes, draw the graphs of* $y = 2x^2 + 5x - 3$ *and* $y = x + 1$ *for* $-4 \leq x \leq 1$ *using intervals of* ***0·5*** *(use a scale of* ***2cm : 1 unit*** *on the* ***x-axis*** *and* ***1cm : 1 unit*** *on the* ***y-axis)***

(b) *Use your graphs to solve the equation* $x^2 + 2x - 2 = 0$

7. (a) *Copy and complete the table below:*

X	***–4***	***–3***	***–2***	***–1***	***0***	***1***	***2***	***3***	***4***
$x^2 - 2$	***14***							***7***	
$-x^2 + 6$	***–10***							***–3***	

(b) *On the same axes, draw the graphs of* $y = x^2 - 2$ *and* $y = -x^2 + 6$ *for* ***–4 ≤ x ≤ 4.*** *(use a scale of* ***1cm : 1 unit*** *on the* ***x-axis*** *and* ***1cm : 2 units*** *on the* ***y-axis)***

(c) *Use your graphs to solve the equation* $x^2 - 2 = 6 - x^2$

8. (a) *Draw a graph of* $y = x^2 - 2x - 3$ *for* ***–2 ≤ x ≤ 4***
(use a scale of ***1cm:1 unit*** *on both axes)*

(c) *Use your graph to solve the equations:*

(i) $x^2 - 2x - 3 = 0$

(ii) $x^2 - 2x - 1 = 0$

(iii) $x^2 - 3x - 3 = 0$

9. (a) *Copy and complete the table below for the function* $y = 6 + 3x - 2x^2$

X	***–2***	***–1·5***	***–1***	***–0·5***	***0***	***0·5***	***1***	***1·5***	***2***	***2·5***	***3***
$-2x^2$	***–8***		***–2***	***–0·5***			***–2***	***–4·5***	***–8***		***–18***
$3x$	***–6***		***–3***	***–1·***			***3***	***4·5***	***6***		***9***

				5							
6	6	6	6	6	6	6	6	6	6	6	6
Y	–8		1	4			7	6	4		–3

(b) *On the same axes, draw the graphs of* $y = 6 + 3x - 2x^2$ *and* $y = 2x$ *for* $-2 \leq x \leq 3$ *(use a scale of* **2cm : 1 unit** *on the* **x-axis** *and* **1cm : 1 units** *on the* **y-axis)**

(c) *Use your graphs to solve the equation* $6 + x - 2x^2 = 0$

10. (a) *On the same axes, draw the graphs of* $y = 6x + x^2 - x^3$ *and* $y = 4x$ *for* $-3 \leq x \leq 4$ *(use a scale of* **1cm : 1 unit** *on the* **x-axis** *and* **1cm : 2 units** *on the* **y-axis)**

(b) *Use your graphs to solve the equations:*

(i) $6x + x^2 - x^3 = 0$

(ii) $2x + x^2 - x^3 = 0$